

University of Arkansas – Fort Smith
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General Syllabus

MATH 44373 Group Theory

Credit Hours: 3

Lecture Hours: 3

Laboratory Hours: 0

Prerequisite: MATH 44073 Abstract Algebra

Effective Catalog: 2018~2019

I. Course Information

A. Catalog Description

Symmetric, alternating, and dihedral groups, direct sums and direct products, generators and relations, group actions, semi-direct products, the Sylow theorems, Sylow subgroups, nilpotent groups, solvable groups, homomorphism groups, fundamental theorem of finite abelian groups, automorphism groups, general linear groups, matrix representations of groups, and applications of group theory.

B. Additional Information

This course serves as one of the required upper level electives in mathematics for students pursuing the bachelor of science degree with a major in mathematics.

II. Student Learning Outcomes

A. Subject Matter

Upon completion of this course, the student will be able to:

1. Perform computations with elements of S_n and A_n .
2. Determine the symmetry group of a given polygon or polyhedron.
3. Perform computations in direct sums, direct products and semi-direct products of given groups, and distinguish among these concepts.
4. Prove the Sylow theorems and apply them to determine the possible structures of finite groups of a given order.
5. Perform computations involving commutators, and identify the commutator and central subgroups of a given group.
6. Perform computations and write proofs involving centralizers and normalizers
7. Prove whether a given group is solvable, nilpotent, or neither.
8. Perform calculations in homomorphism groups.
9. Prove results about automorphism groups.

10. Prove results involving matrix groups and identify subsets of general linear groups.
11. Exhibit the homomorphism between a group and a matrix representation of that group, and use the matrix representation to perform calculations in the given group.
12. Discuss some applications of group theory in other branches of mathematics, physics, chemistry, and other disciplines.

B. University Learning Objectives

Analytical Skills

Critical Thinking Skills: Students will think and communicate results in highly abstract terms. Students will analyze how definitions and methods of proof apply to individual questions is required at every step. Students will study the algebraic structure of permutations increasing a theoretical understanding of counting principles and their application to other areas of Mathematics.

Communication Skills (written and oral)

Students will argue the correctness of their proposed solutions on every problem. Students will write logical proofs in a narrative format for a majority of questions. Students will present their solutions to the class orally.

III. Major Course Topics

- A. Review of fundamental concepts: groups, subgroups, normality, and factor groups.
 1. Group
 2. Subgroup
 3. Normal subgroup
 4. Factor groups
- B. Symmetric and alternating groups, Cayley's theorem.
 1. Isomorphism
 2. Cayley's Theorem
- C. Group action.
 1. Group acting on sets
 2. The class equation
 3. Burnside's counting theorem
- D. Direct and semi-direct products
 1. Direct product
 2. Semi-direct product
- E. Sylow theory
 1. Cauchy's Theorem
 2. P-Groups
 3. The Sylow Theorem
- F. Solvability and nilpotence.
 1. Finite abelian groups
 2. Solvable groups

- 3. Nilpotence
- G. Groups of homomorphisms
 - 1. Factor groups
 - 2. Group homomorphisms
 - 3. The Isomorphism Theorem
- H. Matrix groups and group representations
 - 1. Matrix groups
 - 2. Symmetry
 - 3. Group representations
- I. Applications of group theory.